

**BACCALAURÉAT GÉNÉRAL
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS**

SUJET 6

**Maths and strategy games
Sequences**

Le sujet comporte une page. L'usage de tout modèle de calculatrice, avec ou sans mode examen, est autorisé.

Many games and pastimes have flimsy analogies with induction, that strange procedure by which scientists observe that some ostriches have long necks and conclude that all unobserved ostriches also have long necks. In poker and bridge, for instance, players use observational clues to frame probable hypotheses about an opponent's hand. A cryptographer guesses that a certain "pattern word," say BRBQFBQF, is NONSENSE, then tests this inductive conjecture by trying the letters elsewhere in the message.

Familiar games such as Battleship and Jotto have slightly stronger analogies with scientific method, but the first real induction game was Eleusis, a card game invented by Robert Abbott.

Eleusis intrigued many mathematicians notably Martin D. Kruskal of Princeton University, who worked out an excellent variant that he described in 1962 in a privately issued booklet, Delphi-a Game of Inductive Reasoning.

In Eleusis and Delphi, a secret rule, specifying the order in which single cards may be played, corresponds to a law of nature. Players try to guess the rule inductively and then (like scientists) test their conjectures.

Source: Martin Gardner: Mathematical circus.

The mathematical association of America. Washington, DC 1992

I. Explain what the text deals with and comment on it.

II. Exercise

In the Towers of Hanoi problem, there are three pegs (posts) and n rings of different sizes. At the beginning of the game, all n rings are all on the first peg, arranged so that the largest is on the bottom, and the smallest is on the top (so the first peg looks like a tower).



- The goal of the game is to end up with all rings on the third peg, in the same order, that is, smallest on top, and increasing order towards the bottom. But, there are some restrictions to how the rings are moved:
- The only allowed type of move is to grab one ring from the top of one peg and drop it on another peg. That is, you cannot grab several rings at one time.
- A larger ring can never lie above a smaller ring, on any post.

1. What is the minimum number of moves required to move:

a) 2 rings;

b) 3 rings.

2. Let T_n be the minimum moves required to move n rings from one peg to the other. Then, one can prove that the sequence (T_n) is such that $T_1=1$ and for all $n>1$, $T_n=2T_{n-1}+1$.

Now, let's consider the sequence defined by $Q_n=T_n+1$.

a. Find T_1 , T_2 , T_3

b. Find Q_1 , Q_2 , Q_3

c. Which conjecture can you make about the sequence (Q_n) ?

d. Express Q_n in terms of n .