BACCALAURÉAT GENERAL EPREUVE SPECIFIQUE DES SECTIONS EUROPENNES MATHEMATIQUES – ANGLAIS

CORRIGÉ DU SUJET 7

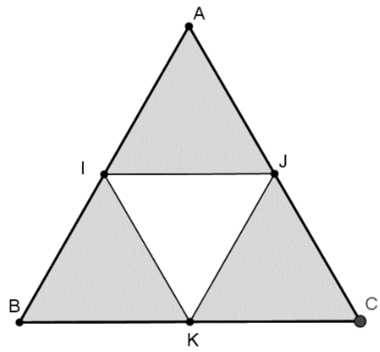
1. This document is a photo from a book entitled SNOW ART by Simon Beck published in 2014. This photo was taken from a plane; it shows an amazing geometric shape made in the snow in the mountains by the artist Simon Beck by stomping the snow with only his snowshoes.
   To realise this huge shape, he started doing an equilateral triangle. Then he used the same pattern for each new triangle: he joined the midpoint of each side of the triangle to get a new triangle.
   After the first iteration, we can see a big white triangle inside the initial one.
   There are 3 triangles left: one at the top, one at the bottom left hand side and one at the bottom right hand side.
   If we continue to use this pattern, the shape we get is called a fractal.

2. We may wonder:

Quelques problématiques envisageables	Des éléments de réponse
a. What is probably the length of the side of the first big triangle ?	3.a.
b. Are all the triangles (both white and "left triangles") equilateral triangles? (we suppose that the first one is equilateral)	3.b
c. How many white triangles are there on the picture ?	3.c
d. What is probably the minimum number of paces Simon Beck had to do to realise this shape ?	3.d
……..	

3. a.
   To build this shape, Simon Beck probably uses the number of paces he does to measure the sides of the triangles (not meters or miles it's too difficult).
   As he divides each side of triangles by 2 and again by 2 and again by 2, and because it's easy to divide by 2, the length of the side of the first triangle is a power of 2. If we look at the shape we can see there are 5 iterations.
   The length of the side of the first big triangle can be $2^7 = 128$ paces which is approximately equal to 128×0.8=102.4 meters (or $102.4\times 1.6 \approx 164$ miles) (if we consider that one pace corresponds to 0.8 m). It seems coherent with the picture.

b. Let's call ABC the initial equilateral triangle whose side is 128 paces.
Let I be the midpoint of [AB], J the midpoint of [AC]
and K the midpoint of [BC].



Using the midpoint theorem in triangle ABC we can state that:
IJ = $\frac{BC}{2} = \frac{128}{2} = 64$ IK = $\frac{AC}{2} = \frac{128}{2}$=64 JK = $\frac{AB}{2} = \frac{128}{2}$=64
IJ = IK = JK = 64 therefore triangle IJK is an equilateral triangle.
And all the triangles we can see on the picture are equilateral
triangles.
Using the same way, we can prove that each triangle we get is an equilateral triangle

c. Let's call u_n the number of triangles left when one triangle is drawn at the n^{th} iteration.
We have $u_1 = 1$ (the big triangle)
Using the pattern, at each step there are 3 new triangles left.

So to get the value of u_n, you have to multiply the previous term by 3.
That means: $u_{n+1} = 3u_n$: we recognize a geometric sequence whose common ratio is 3 and the first term is 1.
Thanks to the photo, we can notice there are 5 iterations.
Let's call S the total number of white triangles there are on the picture.
$S = u_1 + u_2 + \ldots. +u_5$
$S = \frac{1-3^5}{1-3} = 121$ (we can check the result on the picture)

d.Let's call v_n the length of one side of a « left » triangle.
At each iteration the length is divided by 2 (or multiplied by $\frac{1}{2}$) that means : $v_{n+1} = \frac{1}{2} v_n$
(v_n)is a geometric sequence whose common ratio is $\frac{1}{2}$ and the first term v_1 which is the length of the side of the big triangle.

The minimum of paces (called M) Simon Beck has to do to realise this shape is equal to the sum of the perimeters of the white triangles we can see on the picture.
v_n is the length
There is 1 triangle whose length is v_1 . There are 3 triangles whose length is v_2
There are 9 triangles whose length is v_3 …
$v_{n+1} = \frac{1}{2} v_n$ so $v_n = v_1 \times 0.5^{n-1}$ (v_1 = 128 paces)
So $M = 3(v_1 + 3v_2 + 9v_3 + 27v_4 + 81v_5)$
$M = 3(v_1 + 3v_1 \times 0.5 + 9v_1 \times 0.5^2 + 27v_1 \times 0.5^3 + 81v_1 \times 0.5^4)$
$M = 3v_1(1 + 3\times 0.5 + 9\times 0.5^2 + 27\times 0.5^3 + 81\times 0.5^4)$ v_1 = 128 paces

M = 5064 paces ($\approx 4051m \approx 4km$) . But it's clearly more than that.