BACCALAURÉAT GENERAL
EPREUVE SPECIFIQUE DES SECTIONS EUROPENNES
MATHEMATIQUES – ANGLAIS

CORRIGÉ DU SUJET 12

1. $u_0 = 100$, $u_1 = 10$, $u_2 = 1$ and $u_3 = 0.1$
2. $u_n = 10^{2-n} = 100.\left(\frac{1}{10}\right)^n$. ($u_n$) is a geometric sequence, its common ratio is 0.1 and its first term is $u_0 = 100$
3. $\lim\limits_{n\to\infty} u_n = 0$ because (u_n) is a geometric sequence which common ratio is strictly between -1 and +1.
4. Zeno's paradox can be easily solved if we understand that the distance covered by the tortoise (and Achilles) after an infinite number of steps is a finite number, which is something the ancient Greeks did not know.

   Using a sequence, we can easily calculate the limit of the total distance covered by the tortoise. If the finish line is beyond this limit, there is no value of n for which the tortoise will reach it, and thus, for any value of n, the tortoise will be ahead of Achilles.

   If we want to calculate the distance covered by the tortoise after n steps (including the head start) using the sequence (v_n).

   $v_n = 100 + 10 + 1 + 0.1 + 0.01 + \ldots + 100.\left(\frac{1}{10}\right)^n = 100.\sum_{i=0}^{n}\left(\frac{1}{10}\right)^i$

   (v_n) is a geometric series, therefore: $v_n = v_0.\frac{1-r^{n+1}}{1-r} = 100.\frac{\left(1-\left(\frac{1}{10}\right)^{n+1}\right)}{1-\frac{1}{10}}$

   Therefore $\lim\limits_{n\to\infty} v_n = 100.\frac{1}{1-\frac{1}{10}} = \frac{1000}{9} \approx 111.1111$

   After an infinite number of steps, the total distance covered by the tortoise (including the head start) is $\frac{1000}{9}$.

   Thus, if the length of the race is greater than $\frac{1000}{9}$, then for any value of n, the tortoise will be ahead of Achilles.

   Moreover, if the length of the race is lower than $\frac{1000}{9}$, the tortoise will actually win the race.